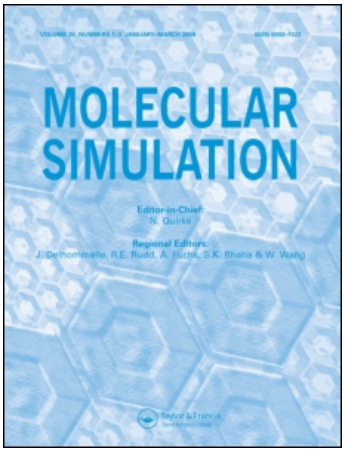

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Modelling Viscous Fingering on a Parallel Computer

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MODELLING VISCOUS FINGERING ON A PARALLEL COMPUTER

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Diffusion limited aggregation (DLA) has proved very successful in modelling systems which display fractal characteristics, like viscous fingering. However, by nature, such simulations are very processor intensive, requiring large amounts of processor time even for relatively small models. We have performed simulations of viscous fingering on the NCUBE parallel computer which has hypercube architecture. We find that, as long as the number of processors used is much less than both the total number of walkers released and the overall dimensions of the model, the fractal dimensions obtained using serial and parallel algorithms give similar results whilst achieving a considerable speed-up in the parallel implementation. An average fractal dimension of 1.71 was obtained along with a speed-up of 106 (in the best case) and 83% efficiency using 128 processors.

KEY WORDS: DLA, viscous fingers, hypercube

1 INTRODUCTION

The unstable displacement that occurs when a fluid of low viscosity is injected into a fluid of high viscosity is termed viscous fingering. The initially straight fluid-fluid interface breaks up into a structure that resembles the fingers of a glove. The leading fingers continue to branch out in time, suggesting that these structures are fractal in nature. Viscous fingering has been observed in enhanced oil recovery when water is used to displace oil trapped in a porous medium. In the laboratory, this phenomenon has been studied in a Hele–Shaw cell which is used to simulate two-fluid flow in a porous medium [1]. In recent years there have also been attempts to model viscous fingering using computational techniques. Paterson [2] showed that fluid displacements in porous media bore a strong resemblance to fractals generated by diffusion limited aggregation (DLA). It has since been shown that viscous fingers show fractal characteristics principally in areas away from steep pressure gradients, i.e., away from injectors (sources) and producers (sinks).

Because of the dramatic effect viscous fingering can have on the efficiency of oil recovery, it is important to study the factors which determine the growth of the fingers and subsequent clustering. By performing computer simulations, it is possible to

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determine the effects of fluid density, viscosity, pressure, etc. on viscous fingering. To serve any useful purpose, these simulations have to be performed on large scale grids and for various combinations of initial parameters. Hence, it is important to design a scheme that is both accurate and computationally fast. It is clear that parallel processing is the only way to achieve high efficiency simulation of realistic phenomena.

The DLA model [3] originally introduced to simulate soot particles is the discrete version of the Langer model [4] for dendritic growth and similar to the model proposed by Sawada *et al.* [5] to describe dielectric breakdown. Thus DLA could be used to simulate a range of phenomena displaying fractal characteristics. The DLA as proposed by Witten and Sander [3] is ideally suited to execution on a serial processor. In this paper, we present an algorithm for implementation on a parallel processor which produces results similar to that obtained with a serial algorithm but with considerable speed-up. Below we outline the DLA model and describe the algorithm used in the calculations. The results are then analysed in terms of the fractal dimension of the finger structure, the CPU time and the efficiency of the calculation.

2 THE DLA MODEL

A two dimensional fractal structure is built by placing a seed particle at the centre of a square grid. A second particle is then introduced into the grid from a corner and allowed to random walk on the grid. If it moves to a site adjacent to the seed particle, it remains at that point forming a two particle cluster. If, however, it does not reach an adjacent site before a fixed number of time steps, it is discarded and another particle is introduced into the grid. To minimise the computational effort, we use a quarter-of-a-five-spot (QFS) type of grid [2] to perform the simulations. The QFS technique requires the injection of particles at one corner of the grid and the seeding at the opposite corner. The modelling of a continuum can be achieved by either transmission or reflection of the particles at the boundaries. In the Paterson model particles were reflected at the boundary whereas in our approach the particle reaching a boundary is transmitted to the face reflected in the leading diagonal. Hence, a particle exiting the west boundary will re-enter the grid from the south boundary, etc. Thus, by successively rotating the grid through ninety degrees and matching it with the original, a full radial model can be simulated. This process is repeated for a large number of particles and the resulting cluster displays a fractal behaviour (see Figure 1). The final site of each particle depends on the number of particles released at any instant minus one.

3 A PARALLEL ALGORITHM FOR DLA

The parallel algorithm devised was specific to the NCUBE1 which is a MIMD local memory system. To achieve maximum efficiency, each node processor was initialised with an identical grid, the seed being placed at a corner, and is provided with particles by the host processor which it releases from the opposite corner to the seed point. A particle is allowed to execute a random walk around the grid until it either exceeds the fixed number of time steps, in which case a request for another particle is sent to the host, or it clusters with an already stationary particle or the seed. In this latter case, the particle number and its position is sent to all the other node processors which update their grids.

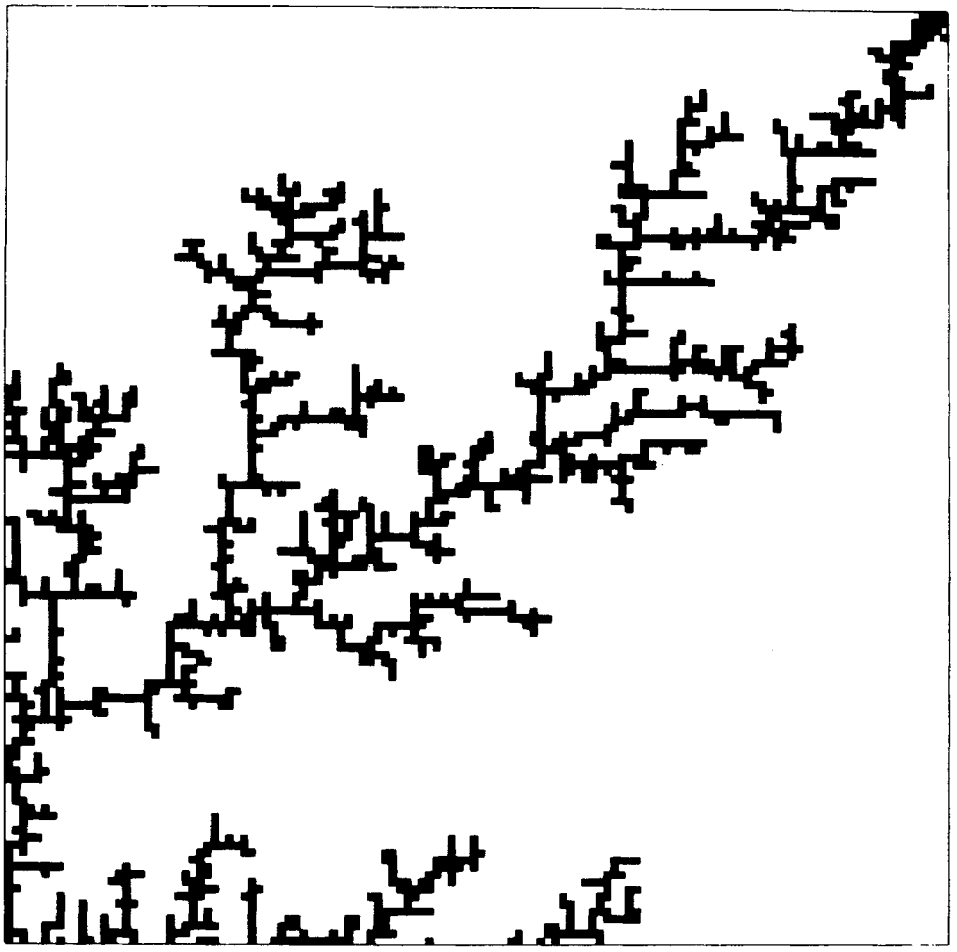

Figure 1 DLA structure obtained using the serial algorithm for a 128 × 128 QFS type grid.

It should be noted that in this model, the active particle on each node depends on the total number of particles released at any one instance minus N, where N is the number of processors. Thus the parallel algorithm can be considered a good approximation to the serial version only if the total number of particles to be released is much greater than the number of processors. Also the number of processors must be less than the length of the leading diagonal of the grid.

4 RESULTS

4.1 Fractal Structure

The simulations were carried out for a maximum of 200 000 particles on a grid of side 128 units. The parallel algorithm was implemented on 1, 4, 16 and 128 node processors and a comparison of the results obtained together with that for the serial version

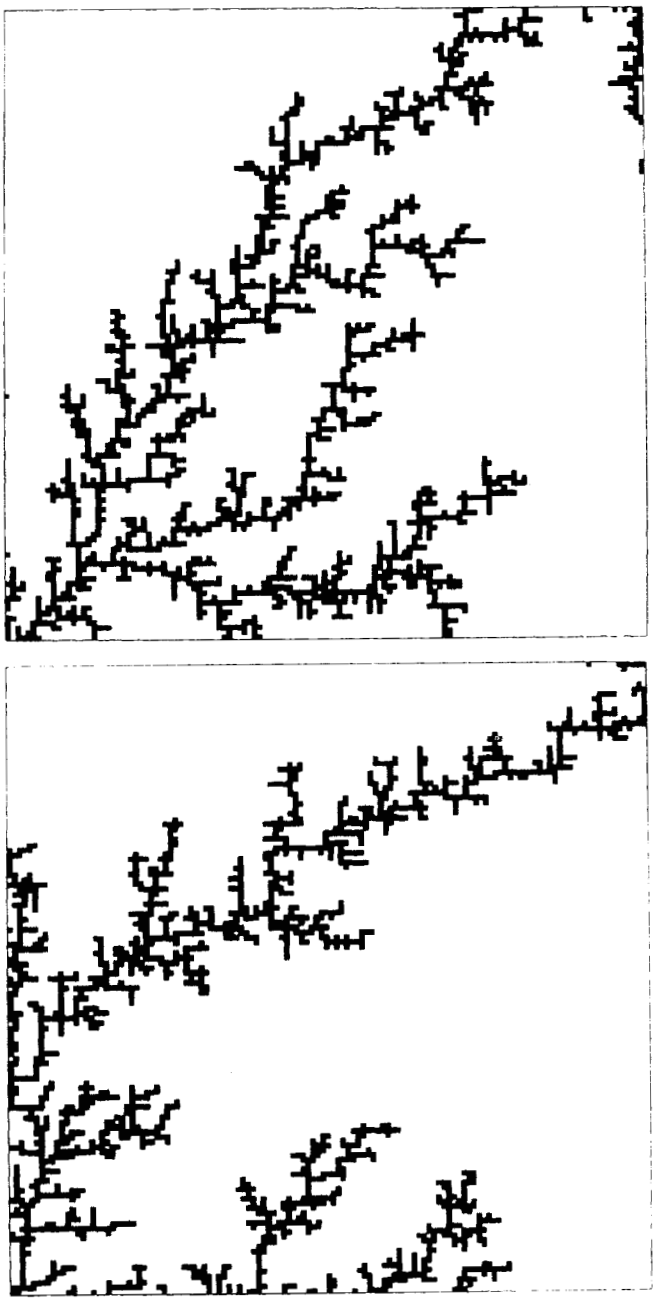

Figure 2 DLA structure obtained using the parallel algorithm for a 128 × 128 QFS type grid with (a) 4 processors and (b) 128 processors.

is given below. Although the fractal structures were different in detail (Figures 1 and 2), a detailed analysis of the fractal dimension shows all simulations were in good agreement with each other and with other calculations and experiment. This suggests that DLA is a good representation of viscous fingering and the parallel algorithm gives a good description of the process.

4.2 Fractal Dimension

To determine the fractal dimension, we use the radius of gyration method [6] and construct a circular shell of radius r and thickness dr (the thickness of an individual particle). The amount of matter enclosed in the shell is proportional to $2\pi r^{D-1}dr$ where D is the fractal dimension. Thus the area, A, occupied by the particles within the shell $2\pi r^2 dr$ is proportional to r^D. Hence a plot of ln(A) against ln(r) yields D as the slope. In Table 1 the fractal dimension for the various simulations is given together with the results of Witten and Sander [3] and Nittmann *et al.* [6]. It may be observed that the parallel algorithm gives a slightly lower value for D than the serial version.

4.3 Speed-up and Efficiency

Taking T_s to be the time taken for the serial algorithm on a serial computer, and T_p to be the time taken for the parallel algorithm, the speed-up for N processors is defined as

$$S_N = T_s/T_p$$

The efficiency for N processors is given by

$$E_N = S_N/N$$

From the results shown in Table 2, it can be seen that as the number of processors is increased, the efficiency remains approximately constant resulting in substantial speed-up for a large number of processors. An original execution time of 10 h for the single processor was reduced to about 5 min when 128 processors were utilised.

5 CONCLUSION

We have shown that a parallel algorithm based on N particles executing random walks simultaneously on N processors gives a fractal structure very similar to that obtained with the particles undergoing sequential walks. However the speed-up increased almost linearly with the number of processors used. From a physical point of view, the parallel algorithm may be the more realistic as particles do move in a parallel fashion rather than sequentially. However, it ought to be noted that although some problems are easily parallelised on a hypercube architecture, others may not be open to such solutions. In our attempts to construct parallel algorithms to produce Mandelbrot and Julia fracta structures, we found that the efficiency was reduced dramatically with increased number of processors and a speed-up of only two was obtained.

Acknowledgements

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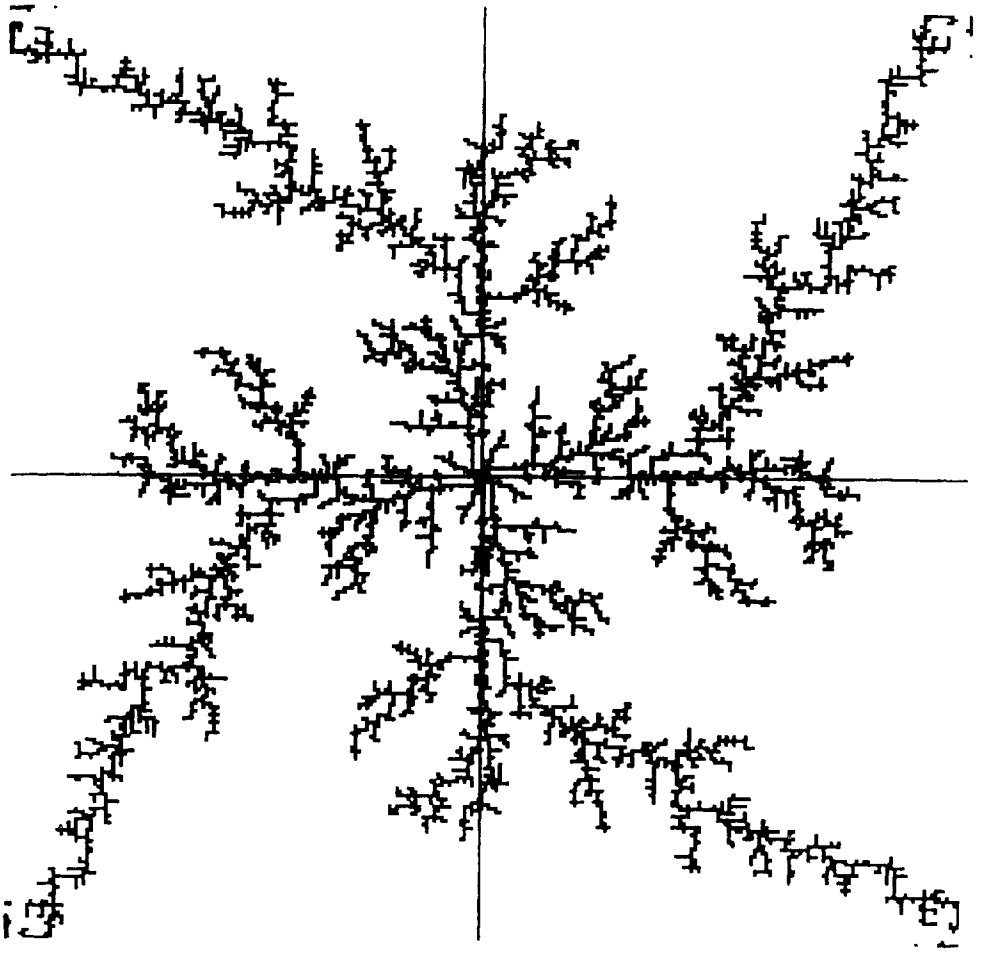

Figure 3 Full radial model of cluster obtained with the parallel algorithm and 128 processors.

Table 1 A comparison of the fractal dimension obtained from our simulations with other calculations and experiment

Simulation	*No of processors*	*Fractal Dimension*
Serial	1	1.71
Parallel	1	1.83
Parallel	4	1.70
Parallel	16	1.64
Parallel	128	1.65
Ref 3		1.71
Expt [6]		1.70

Table 2 Speed-up and efficiency as a function of the number of processors

N	S_N	E_N
4	3.53	88.25
16	13.25	82.81
128	106.00	82.81

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